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BRIEF NOTES

Strain Rates of Micropolar Continua Equivalent to Discrete Systems

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1. Introduction

Assemblies of rigid blocks can be perceived as fine models of rocky materials, masonry materials and any other composite material made of elements essentially undeformable with respect to the matrix. However, it is often convenient to model such materials as continua endowed with structure in the sense specified in [**?**]. In an earlier work, a continuum constitutive model of a discrete system has been derived in the framework of micropolar linear elasticity [**?**]. In particular, the equivalence of the mechanical power of the continuum and the discrete model for any virtual velocity fields led to the identification of the constitutive functions of the continuum stress measures. In the present note, this integral criterion of equivalence is adopted to identify the strain rates of a micropolar elastic–perfectly plastic continuum equivalent to rigid particles systems with no-tension constraints; the correspondence between the power of the continuum and of the discrete model is required for any virtual stress field.

2. Identification of the Plastic Strain Rates of the Equivalent Continuum

Let $\mathbf{w}$ and $\mathbf{W}$ be, respectively, the velocity vector and the angular velocity skewsymmetric tensor of a material point of a Cosserat continuum. The linearized strain measures are defined as $\mathbf{U} = \operatorname{grad}\mathbf{w} - \mathbf{W}$ and $\mathsf{U} = \operatorname{grad}\mathbf{W}$. The mechanical density power of a linear elastic–plastic Cosserat continuum can be written (e.g. [**?**]) as

$$\mathcal{P} = \mathbf{S}\cdot(\mathbf{U}^{\mathrm{e}} + \mathbf{U}^{\mathrm{p}}) + \tfrac{1}{2}\mathsf{S}\cdot(\mathsf{U}^{\mathrm{e}} + \mathsf{U}^{\mathrm{p}}), \tag{1}$$

where the second-order tensors $\mathbf{U}^{\mathrm{e}}$ and $\mathbf{U}^{\mathrm{p}}$ and the third-order tensors U^{e} and U^{p} represent the elastic ('e') and the plastic ('p') part of the strain and of the micro-strain rates, while $\mathbf{S}$ is the second-order linearized stress tensor and S the third-order linearized micro-stress tensor.

Considering a representative part ('module') $\mathcal{M}$, of volume $\mathcal{V}$, of the discrete system, the mean power formula can be written as

$$\pi = \frac{1}{V}\sum_{c}\{\mathbf{t}_c\cdot(\mathbf{w}_c^{\mathrm{e}} + \mathbf{w}_c^{\mathrm{p}}) + \tfrac{1}{2}\mathbf{C}_c\cdot(\mathbf{W}_c^{\mathrm{e}} + \mathbf{W}_c^{\mathrm{p}})\}, \tag{2}$$

The apex ‘T’ indicates the major transposition of a tensor. Now it can be observed that through equations (**??**), the yield functions are functions of the regular fields $\mathbf{S}$ and S and can be differentiated with respect to them:

$$
\begin{aligned}
\frac{\partial \mathcal{F}_c}{\partial \mathbf{S}} &= \frac{\partial \mathcal{F}_c}{\partial \mathbf{t}_c}\frac{\partial \mathbf{t}_c}{\partial \mathbf{S}} = \frac{\partial \mathcal{F}_c}{\partial \mathbf{t}_c}\mathbf{K}_c\mathbb{A}^{-1} \otimes (g^a - g^b),\\
\frac{\partial \mathcal{F}_c}{\partial \mathsf{S}} &= \frac{\partial \mathcal{F}_c}{\partial \mathbf{C}_c}\frac{\partial \mathbf{C}_c}{\partial \mathsf{S}} = \frac{\partial \mathcal{F}_c}{\partial \mathbf{C}_c}\mathbb{K}_c\mathbb{D}^{-1} \otimes (g^a - g^b),
\end{aligned}
$$

where $\partial \mathcal{F}_c/\partial \mathbf{S}$ is a third-order tensor and $\partial \mathcal{F}_c/\partial \mathsf{S}$ is a fourth-order tensor.

Therefore, taking into account the symmetries of the tensors $\mathbf{K}_c$, $\mathbb{K}_c$, $\mathbb{A}$ and $\mathbb{D}$, the strain rates are identified as

$$
\mathbf{U}^{\mathrm{p}} = \frac{1}{V}\sum_c \left(\frac{\partial \mathcal{F}_c}{\partial \mathbf{S}}\right)^{\mathrm{T}} \lambda_c, \qquad \mathsf{U}^{\mathrm{p}} = \frac{1}{V}\sum_c \left(\left(\frac{\partial \mathcal{F}_c}{\partial \mathsf{S}}\right)^{\mathrm{T}}\right)^{\mathrm{t}} \lambda_c, \tag{9}
$$

where the symbol ‘t’ stands for the minor right transposition.

The constitutive functions for the plastic strain measures of the equivalent micropolar continuum are determined when the geometry of the discrete model and the yield domain of its contact points are known. Consequently, it is possible to construct, using a standard procedure, the macroscopic elastic–plastic tensor and then solve the continuum problem.

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