STONE ASSEMBLIES UNDER IN-PLANE ACTIONS
COMPARISON BETWEEN NONLINEAR DISCRETE APPROACHES

C. Baggio[1] and P. Trovalusci[2]

1 ABSTRACT

Brick-block masonry, perceived as a jointed assembly of prismatic particles in dry contact, is modeled as a discrete system of rigid blocks interacting by compressive and frictional actions. The interactions are modeled as non-linear springs and as nonlinear unperfected constraints. Numerical analyses are undertaken by two different approaches: a non linear finite element analysis (FE) and a non-standard limit analysis (LA). Some theoretical aspects concerning the two models are discussed in detail. The influence of various parameters in the two cases is pointed out with reference to a sample test. The advantages and the limits of the FE and of the optimization procedures are made evident by comparing experimental and numerical results for typical samples. The effectiveness in the simulation of the actual masonry behaviour using a discrete approach, is further evaluated by comparing the obtained results with those of a distinct element approach (DE).

2 INTRODUCTION

In this work we investigate the mechanical behaviour of two-dimensional block masonry walls subjected to the static action of in-plane loads. In the last twenty years the modeling of masonry media became an argument particularly appealing for many researchers. Because of the complex phenomenology of the object, the problem is far from being solved in a general way. Nevertheless, the building practice often requires technical solutions for very particular problems. That incentives the modeling by means of specific numerical investigations.
Our attention is focused on an important class of ancient masonry made by stones dry assembled together in various dispositions. The main mechanical features of this kind of assemblies are the substantial coherence and undeformability of the blocks with respect to the deformability of the joints, the lack of tensile strength together with the frictional

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properties of the interfaces, the geometry of the units and their texture. In several works the influence of the former two aspects were pointed out [1, 2]. One of the purposes of the analyses here performed is to make evident the importance of the latter two features. We observed that the size, the arrangement and the orientation of the units strongly influence the behaviour of brick masonry as well as the no-tensional and the frictional properties of the interfaces. In order to take into account both the geometrical and the mechanical properties, the masonry assembly is here regarded as a discrete system of rigid blocks interacting either by non-linear springs or by contact surfaces incapable to carry tension and resisting to sliding by friction. With reference to the two-dimensional case, we employed two different approaches. First, in the framework of a non-linear Finite Element analysis, the blocky material is modeled by rigid constraints for the units and by *gap* elements for the joints, like a "jointed system" in rock mechanics [3], Secondly we develop a computer procedure for a non standard limit analysis, resorting the analogy with the nonassociated flow rules [4, 5, 6].
We have already implemented two of the mentioned procedures obtaining satisfactory results which have encouraged further applications [7]. Here we analyse the two procedures from a theoretical and a practical point of view comparing the respective computational complexity, the constitutive prescriptions required, the parameters to define for the analyses, the effectiveness in numerical simulation of the experimental results. In order to make evident the suitability of the various methods available within the framework of statical analyses, we also compare the results obtained using the two procedures together with the results of a Distinct Element analysis effected on the same samples under statical load condition [8]. Our endeavour is to individuate the most effective method to evaluate the safety of the block structure, with particular regard to the seismic actions and, at the same time, to provide some criteria for modeling the blocky materials properly accounting for bricks shape, size and arrangement.

3 A SAMPLE TEST

A single rigid block resting on the ground is subjected to the force f of components $f_1 = \lambda w$ and $f_2 = w$, with w the self weight and λ a non negative factor. The contact surface is divided into two parts showing different coefficients of friction, $tg\phi^p$ and $tg\phi^q$. The analysis tends to compute the value $\lambda_c w$ of the horizontal force component which overcomes the resistance of the supports. This problem is suitable to introduce in a direct and simple manner the outstanding features of the two above mentioned approaches (Fig. 1a).
Firstly, we model the supports with four couples of non linear springs (*gap* elements) acting in the direction normal and tangential to the contact surfaces (Fig. 1b). The constitutive functions for the contact actions, $\mathbf{r}^{p_\alpha}$, on a gap element p_α ($\alpha = 1, 2$) are shown in Fig. 1d. Note that, due to the Coulomb friction, the shear behaviour is softening when the compressive forces are relaxed. The problem can be easily solved without resort to a computer program. Assuming for each spring, in the elastic range, the same normal and tangential stiffness, k_n and k_t respectively, the vertical actions assume the following values: $r_2^{p_1} = -\frac{w}{4}(1-\lambda)$; $r_2^{p_2} = -\frac{w}{4}$; $r_2^{q_1} = -\frac{w}{4}$; $r_2^{q_2} = -\frac{w}{4}(1+\lambda)$; the horizontal actions, equal each other until behaviour remains in the linear elastic range, can withstand as a whole an ultimate total force of intensity $\lambda_c w = |tg\phi^p(r_2^{p_1} + r_2^{p_2}) + tg\phi^q(r_2^{q_1} + r_2^{q_2})|$, because of the elastic plastic behaviour of the tangential springs. Hence $\lambda_c = 0.476$. It is evident that the collapse load multiplier corresponding to the overturning mechanism is greater than 0.476 and then this mechanism is not expected for this problem. We obtain a unique ultimate solution that depends only on the position of the gap elements, whereas neither stiffness constants values nor their ratio influence the result.
Secondly, we consider not perfect constraints which inhibit the penetration between the

contact surfaces and the sliding by friction (Fig. 1c). Since only the equilibrium equations are available, λ_c depends on three unknown reactions. Within the framework of limit analysis, if the collapse mechanism shown in Fig. 1c is assumed, the statically admissible set of contact actions that the block exerts on the two supports is such that $r_1^p = |r_2^p tg\phi^p|$; $r_1^q = |r_2^q tg\phi^q|$, and that the balance is satisfied. From the balance of forces, $\lambda_c = (tg\phi^p - tg\phi^q)w^{-1}r_2^p + tg\phi^q$, with the four unknowns r_2^p, r_3^p, r_2^q, r_3^q which must satisfy the moment balance equation. As expected, in presence of Coulomb friction, the ultimate load factor is not unique.From a practical point of view, a way of overcoming this indeterminacy is to determine the minimum value in the class of the admissible collapse multipliers; value which is sufficient to ensure the structural safety. The minimum value of the collapse multiplier for this mechanism is achieved for $r_2^p = 0$; $\lambda_c = tg\phi^q = 0.4$. By performing simple calculations we also obtain the maximum value of the ultimate factor which still satisfies the balance and the yield conditions for all the contact actions: $\lambda_c = tg\phi^p(1 + 0.5(tg\phi^p - tg\phi^q))^{-1} = 0.545$.

It is worth to note that the analysis with deformable supports offers an unique collapse load factor comprised between the minimum and the maximum factor evaluable by limit analysis. Here below we briefly recall the distinctive features of the two procedures from a theoretical point of view. In Section 6 we describe the devices adopted in the two cases to solve complex bricks structures under in-plane static loads.

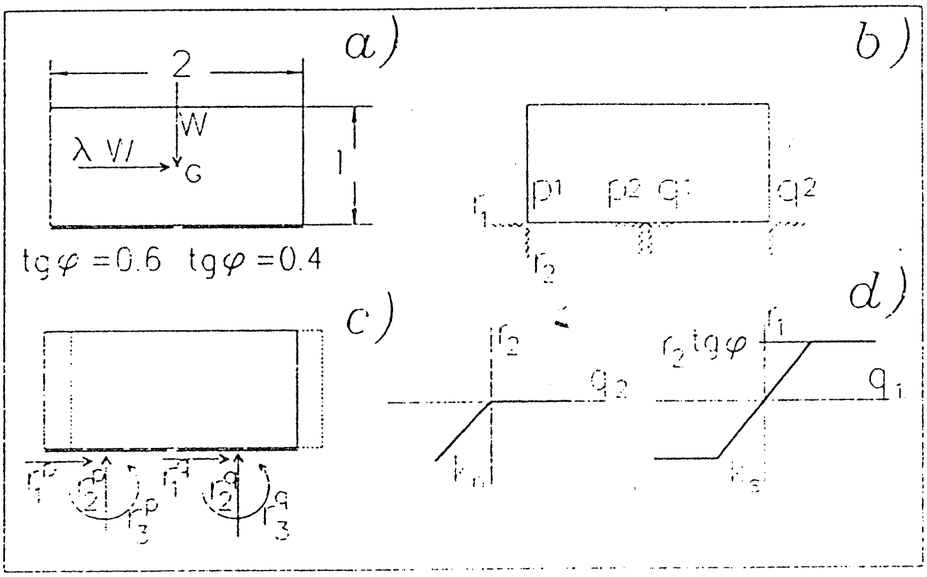

Fig. 1

4 THE MODEL WITH NON LINEAR SPRINGS

As for the sample above, we firstly consider block assemblies made of distinct elements interacting through links unable to carry tension and resistant to sliding by friction. Each link between two blocks is represented with a couple of springs, with stiffness normal and tangential to the contact surface. Due to the relative displacement between two blocks, interactions forces arise. As before, the constitutive functions are piecewise linear: no-tensional elastic for the normal spring and elastic-plastic, with the yield threshold defined

based on Coulomb friction law, for the tangential one. To analyse brick masonry walls resorting this fine modeling, we exploit the general purpose Finite Element code (ANSYS version 5.0) which includes contact elements suitable to describe the joints behaviour.
To drastically condense the degrees of freedom of the structure analysed, we consider the blocks as rigid. For ancient masonry, but also for ordinary masonry walls subjected to lateral body forces, the fractures occurs mainly along the joints and then this hypothesis is not restrictive. In detail a set of nodes, linked by rigid constraints, describe each block, while the contacts surfaces are discretised by a sequence of *gap* elements arranged as in Fig. 1b. The gap element is a two nodes element linked by a couple of nonlinear and nonconservative springs described above. The behaviour of this kind of element is well known. Here we recall that each node has only the translational degrees of freedom; the two nodes can be initially coincident and there are not shape functions for the displacements; only small displacements and small deformations are allowed. Moreover, the interactions are described by contact forces, $\mathbf{r}^p$, and the element stiffness, apart from the stiffness constant, k_n and k_t, depends on the element status, which can be 'closed' – both normal and tangential forces are transmitted across the gap – 'closed and slided' – since the Coulomb law holds, the tangential force carried depends on the compressive one – or 'open'– no interactions are allowed between the two contact points.

5 THE MODEL WITH NONLINEAR CONSTRAINTS

Dismissing the hypothesis of joints deformability, which affords some uncertainties about the determination of the stiffness material parameters, masonry is also described as a system of n rigid blocks directly interacting through m contact surfaces unable to carry tension and resistant to sliding by friction.
The solution of this problem, obtained by means of an optimization procedure, require the description of some details. Each block, $\mathcal{A}$, is subjected to the static action of a force and a couple represented by the vector $\mathbf{f}^a = \sum_{\alpha=1}^{2} f_\alpha^a \mathbf{e}_\alpha + f_3^a \mathbf{e}_3$, where $\{\mathbf{e}_i\}$ is an orthonormal base in $\mathcal{R}^3$ having $\mathbf{e}_1$ and $\mathbf{e}_2$ respectively parallel and normal to the contact surface. The p-th interaction between two adjacent blocks, $\mathcal{A}$ and $\mathcal{B}$ is also described by a force and a couple: $\mathbf{r}^p = \sum_{\alpha=1}^{2} r_\alpha^p \mathbf{e}_\alpha + r_3^p \mathbf{e}_3$ $(\mathbf{r}^p = \mathbf{r}^{pa} = -\mathbf{r}^{pb})$.
Denoted $\mathbf{u}^a = \sum_{\alpha=1}^{2} u_\alpha^a \mathbf{e}_\alpha + u_3^a \mathbf{e}_3$ the infinitesimal generalized displacement of a block – with $\sum_{\alpha=1}^{2} u_\alpha^a \mathbf{e}_\alpha$ the translation of the block center $\mathbf{g}^a$ and $u_3^a \mathbf{e}_3$ the block rotation – the generalized strain measures $\mathbf{q}^p$ at interface p between two blocks are defined by the sum of the relative translations and the relative rotations

$$\sum_{\alpha=1}^{2}(u_\alpha^b - u_\alpha^a)\mathbf{e}_\alpha + \mathbf{e}_3 \times (u_3^b(p^b - g^b) + u_3^a(p^b - g^a)) \quad \text{and} \quad (u_3^b - u_3^a)\mathbf{e}_3$$

where p^a and p^b are the positions of two points on the middle of the contact edge on the blocks $\mathcal{A}$ and $\mathcal{B}$ respectively. We assume that in the reference configuration this points coincide. Then, the kinematical compatibility for the whole system of m interfaces and n units can be written in the form

$$\mathbf{q} = \mathbf{B}\mathbf{u}\,, \tag{1}$$

where B is a linear transform $\mathcal{R}^{3n} \to \mathcal{R}^{3m}$ and, chosen the orthonormal bases $\{\mathbf{e}'_i\} \in \mathcal{R}^{3n}$ and $\{\mathbf{e}''_i\} \in \mathcal{R}^{3m}$, $\mathbf{u} = \sum_{i=1}^{n} \sum_{j=1}^{3} \mathbf{e}'_{3(i-1)+j} \otimes \mathbf{e}_j \mathbf{u}^i$ and $\mathbf{q} = \sum_{i=1}^{m} \sum_{j=1}^{3} \mathbf{e}''_{3(i-1)+j} \otimes \mathbf{e}_j \mathbf{q}^i$. If the vector subspace of the solution of the homogeneous system – the 'kernel' of B – has dimension zero, as for the structures here considered, (1) can assume the form

$$\mathbf{q}_1 = \mathbf{B}_1\mathbf{u}\,, \qquad \mathbf{q}_2 = \mathbf{B}_2\mathbf{u} \tag{2}$$

where the kinematic matrix (B_1) has the maximum rank (3n), $q_1 \in \mathcal{R}^{3n}$ is the vector of the free strains and $q_2 \in \mathcal{R}^{3(m-n)}$ the vector of the linearly dependent strains. In this case exists a unique solution

$$u = A_0 q_1 \; : \qquad\qquad q_2 = A q_1 \tag{3}$$

with $A_0 = B_1^{-1}$ and $A = B_2 B_1^{-1}$.
Said $f \in \mathcal{R}^{3n}$ the generalized load vector of the assembly, $f = \sum_{i=1}^{n} \sum_{j=1}^{3} e'_{3(i-1)+j} \otimes e_i f^i$, from the principle of virtual work we obtained the balance equations

$$A_0^T f = A^T r_2 + r_1 \tag{4}$$

Here $r_1 \in \mathcal{R}^{3n}$ and $r_2 \in \mathcal{R}^{3(m-n)}$ are the contact actions which works in the strains q_1 and q_2 respectively.
The constitutive prescriptions for the components of the contact actions on p

$$|r_1^p| \leq tg\phi^p r_2 \,, \qquad |r_3^p| \leq l^p r_2 \qquad \text{with} \qquad r_2^p > 0 \,,$$

where ϕ^p is the angle of friction and l^p the half length of the p-th interface, characterize a rigid-fracturing behaviour of the joints. These relations define four yield functions for the contact actions on p which represent the components of the vector $\hat{y}^p \in \mathcal{R}^4$. The generalized yield domain of the system can be expressed by the inequality

$$\hat{y} = N^T r = N_1^T r_1 + N_2^T r_2 \leq 0 \tag{5}$$

where $\hat{y} = \sum_{i=1}^{m} \sum_{j=1}^{4} e'''_{4(i-1)+j} \otimes e''''_j \hat{y}^i$, using the orthonormal bases $\{e'''_i\} \in \mathcal{R}^{4m}$ and $\{e''''_i\} \in \mathcal{R}^4$. This domain, regular and not strictly convex, is bounded by the piecewise linear surfaces ($\hat{y}_i = 0$, $i = 1, 4m$). In (5) the linear map $N^T := \mathcal{R}^{3m} \rightarrow \mathcal{R}^{4m}$ is such that each vector Ne'''_i is normal to the i-th yield surface; $N_1^T := \mathcal{R}^{3n} \rightarrow \mathcal{R}^{4m}$ and $N_2^T := \mathcal{R}^{3(m-n)} \rightarrow \mathcal{R}^{4m}$.
The components of the generalized strain q can then expressed as linear combination, with non-negative coefficients m_i, of $4m$ elementary strain modes Me'''_i in such a way that:

$$q = Mm \,, \qquad \text{or also} \qquad q_1 = M_1 m \,, \qquad q_2 = M_2 m \tag{6}$$

where $m = m_i e'''_i$, $M := \mathcal{R}^{4m} \rightarrow \mathcal{R}^{3m}$, $M_1 := \mathcal{R}^{4m} \rightarrow \mathcal{R}^{3m}$ and $M_2 := \mathcal{R}^{4m} \rightarrow \mathcal{R}^{3(m-n)}$.
Finally, m must be such that the complementarity condition

$$\hat{y} \cdot m = 0 \tag{7}$$

is satisfied.
The relationships (3-7) characterize the problem of rigid blocks in no-tension and frictional contacts. Within the framework of the "holonomic" perfect plasticity, the same relations govern the problem of a non-standard rigid-plastic discrete (or discretised) materials [9]. Resorting this formal analogy, as is usual in the specific literature, we are interested to determine the collapse load for a masonry structure, under the hypothesis of proportional load with the sole factor $\lambda \geq 0$. In this case the vector f in (4) split in such a way that $f = f_0 + \lambda f_L$.
Due to the presence of non-associative flow-rules (6), the Drucker stability postulate no longer holds and the solution, in terms of contact actions and collapse factor λ_c loses its uniqueness. However, the set of kinematically and statically admissible multipliers is bounded and, in order to verify the structural safety, the lower bound can be determined

as suggested in [4]. From (3-7) and by requiring the work of live load be positive, after some algebra, we derive the following programming problem

$$\begin{aligned} \lambda_c = \min\{\lambda\} \quad : \quad & (\mathbf{AM}_1 - \mathbf{M}_2)\mathbf{m} = 0 & (8) \\ - \quad & (\mathbf{A}_0\mathbf{N}_1)^T(\mathbf{f}_0 + \lambda \mathbf{f}_L) + (\mathbf{N}_2 - \mathbf{AN}_1)^T \mathbf{r}_2 \leq 0 & (9) \\ - \quad & (\mathbf{A}_0\mathbf{N}_1)^T \cdot (\mathbf{m} \otimes (\mathbf{f}_0 + \lambda \mathbf{f}_L)) + (\mathbf{N}_2 - \mathbf{AN}_1)^T \cdot (\mathbf{m} \otimes \mathbf{r}_2) = 0 & (10) \\ & (\mathbf{A}_0\mathbf{M}_1)^T \mathbf{f}_L \cdot \mathbf{m} = 1 & (11) \end{aligned}$$

with the unknowns λ, $\mathbf{r}_2$, $\mathbf{m}$ and the bounds on the unknowns $\lambda \geq 0$ and $\mathbf{m} \geq 0$.

5.1 Limit analysis: computational aspects

Our first concern was to find out a routine directly able to deal with the non-linear constrained minimization problem; some routines pertaining to the class of second-order derivative methods, appeared suitable in our investigation. These are based on the merit function method and employ the augmented Lagrange function as merit function. An iterative procedure searches the solution by a quadratic approximation of the Lagrangian and by a transformation of the sole non-linear constraint (10) in a set of linear constraints. A first attempt to implement one of these routines, based on the automatic numerical evaluation of the gradients of the objective function and of all the constraints by finite differences, did not perform as expected, so we were forced to choose a more sophisticated routine which requires the explicit definition of the gradients. Though appealing from a theoretical point of view and easy to implement using the IMSL library routines, the above discussed formulation of the LA problem gives rise to a severe numerical task since the assemblies object of the study often result cumbersome. Indeed the high number of scalar unknowns - for instance referring to Fig. 3 these amount to 281 - afford a nearly prohibitive numerical problem and the procedure implemented on a computer appeared to perform poorly. The actual difficulty consists in the number of joints among a set of blocks or, in other words, in the degree of statical indeterminacy of the structure. If the number of joints were equal to the number of blocks, the static unknowns $\mathbf{r}_2$ would not exist, the set of equations (8) would vanish and the problem would be only apparently non-linear. In fact, by Eq. (9) the maximum value of λ which satisfies the set of inequalities can be easily detected as the minimum factor for which at least one of the $4m$ yield faces became active $\hat{y}_j = 0$

$$\lambda_c = (\mathbf{v} \cdot \mathbf{f}_o)(\mathbf{v} \cdot \mathbf{f}_l)^{-1}, \qquad \text{with} \qquad \mathbf{v} = (\mathbf{e}'_i \otimes \mathbf{e}'_i)(\mathbf{AN}_1 \mathbf{e}'''_j), \tag{12}$$

If the inequality satisfied is only the j-th, $\hat{y}_k < 0$ for $k \neq j$ ($k = 1, 4m$) and then Eq. (10) implies that the corresponding multipliers m_k must be zero, while (11) requires that $m_j \neq 0$, that is in one of the joints at least a relative displacement occours and the collapse mechanism is readily found. In other words, for a kinematically determined structure to relax one of the constraints implies a solution with one degree of indeterminacy removable from the prescription of positive normalized work of the live load.

This observation suggests a way to cope with the problem. It is well known that non linear programming could sometimes lead to a local minimum instead of the global one, overestimating in turn the collapse multiplier. Instead, if a distribution of contact forces $\mathbf{r}_2$ not far from the true one can be guessed and used as starting values in the minimization, the analysis easily converges to the exact solution. For the masonry walls object of the present work - walls with regular texture subjected to the action of self weight and a proportional horizontal body force - it appears that head joints transmit negligible actions or no actions between the bricks and that, under the overturning moment due to horizontal body force, only one of the two interfaces along each bed joint is loaded.

The trick consist in ordering the components of the kinematical matrix (B) in such a way that the maximum rank matrix (B_1) defines a kinematically determined system made out of all the n blocks and of an equal number of joints guessed as statically active. So, with an opportune numbering of the joints (Fig. 2), the values of the additional static unknowns, components of r_2, can be initially input as zero in order to direct the optimization procedure to the right solution. This approach came out to be successfully, and we started to get completely satisfying results.

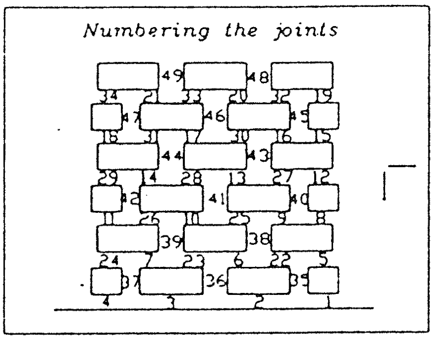

Fig. 2

6 COMPARISON BETWEEN NUMERICAL AND PHYSICAL RESULTS

A 21-bricks wall, sample of a series of experiments on small scale wall models (Fig. 3a), was chosen as test of the effectiveness of the two proposed methods in reproducing failure mechanism and failure load. The physical sample is acted upon only by the self-weight of the blocks; inclination of the testing table thus produces two components of the body force acting along directions parallel and perpendicular to the bed joints. Though limited in the number of elements, this structure shows however all the relevant features of the class of dry-assembled bodies object of the study. Figs. 3b, 3c, 4 show, respectively, computer plots from FE non-linear analysis, from limit analysis and from analysis by the distinct element method [8]. As shown, collapse mechanisms compare each other strictly, whereas the value of the experimental collapse load results about 20% lesser than the computed ones.

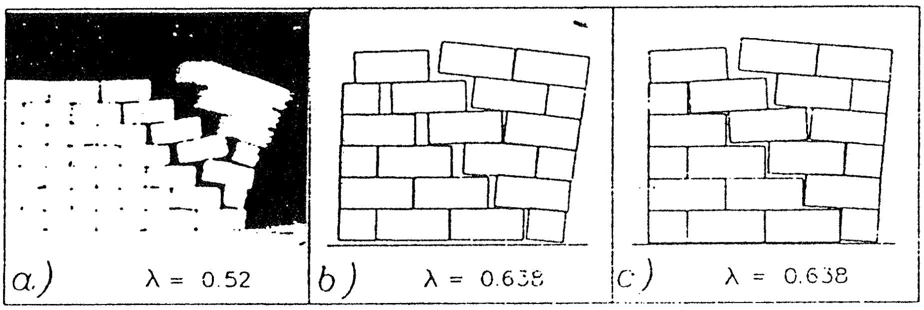

Fig. 3

Since the results of the numerical analyses validate each other, we were forced to conclude that the reduced ultimate strength of the physical model has to be imputed to defects in the planarity of the contact surfaces between the blocks, indeed clearly detected during the tests. Though equivalent from the point of view of correctness and reliability of the results, the three methods highly differ from the point of view of onerousness. In the following table are reported the CPU times requested, on a HP 9720 workstation, to analyse the sample wall and also an evaluation of the work required to prepare input data and to monitor computer runs.

	FE	LA	DE
CPU time (min)	20	15	360
men time	low	medium	high

It is quite evident that the use of DE is not justified, unless a real dynamic problem is involved or analysis with some imperfections must be carried out. Exploiting the advantage of the easier implementation, when evaluating the failure load of a structure under static action, the FE method should be preferred to LA. Moreover, the current version of our LA code cannot handle, in a reliable way, walls made by more then 50 blocks (static and kinematic unknowns sum up to 750 in this case, representing an upper limit for the numerical algorithm). The limit of the two other procedures is not known, but samples made by 220 and 440 blocks were analysed by DE and by FE, respectively. It must be remembered, however, that models with deformable joints, as shown in Section 2 can overestimate the collapse load whereas limit analysis directly searches the lower bound of the failure load. In favour of safety, we use the LA procedure to perform the following structural applications.

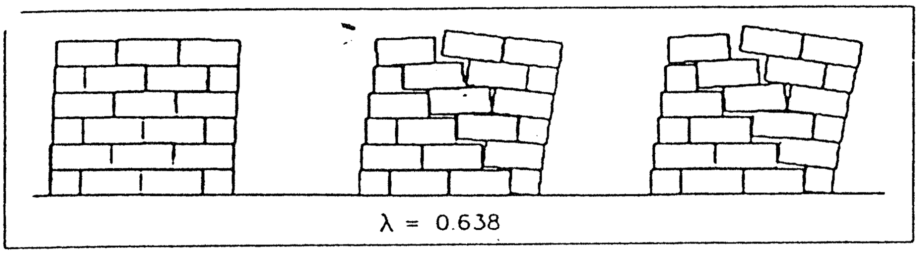

Fig. 4

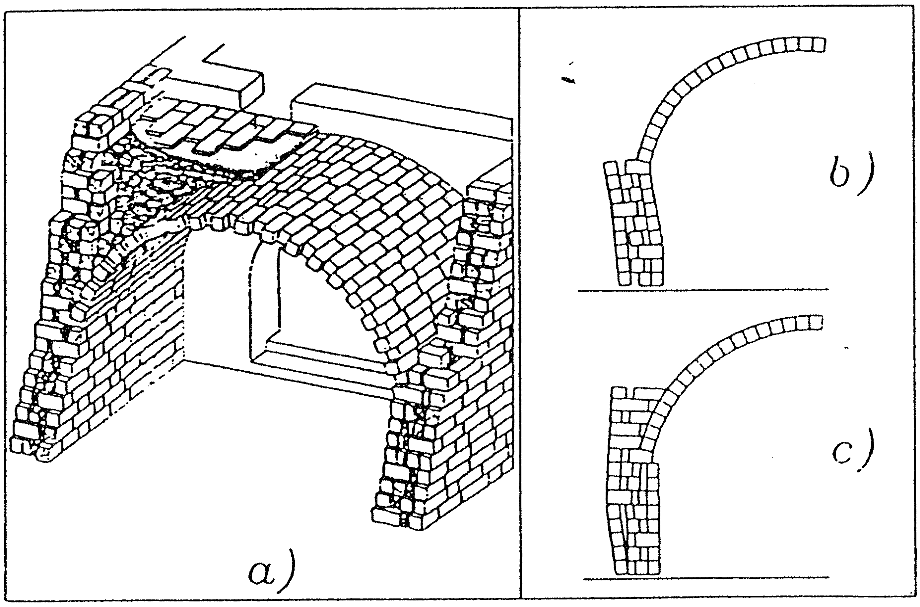

Fig. 5

7 STRUCTURAL APPLICATIONS

A typological analysis independently carried out by a group of researchers on the building techniques of an old dwelling site in southern Italy [10], the so-called "Sassi di Matera", offered the chance of testing the limit analysis approach in evaluating the behaviour of real masonry structures. The question was how walls made by small prismatical blocks connected by a poor and weathered mortar, with few bonding stones and a number of cavities could sustain one or two levels of vaults (spanning 6 meters or more), 25 cm in thickness, made by the same poor masonry assembly (Fig. 5a).
The 3D texture of the wall must be transformed into a plane simplified model acted upon by the vertical dead weight, f_o, while, to evaluate the maximum service load that the vault can sustain, vertical downward forces are applied on the arch partly as live load, λf_L, and partly as dead load. That is necessary to prevent the voussoirs from slipping under zero load, as it happens when all the weight is applied as live force. A first result of the analysis is shown in Fig. 5b in terms of collapse mechanism; nonetheless the structural performance seems to be poor: bonding stones do not prevent the breaking up of the wall under thirty per cent, only, of the total self weight of the vault. Fig. 5c shows a better but not satisfactory result obtained by increasing weight on the wall itself (structure collapses under ninety per cent of the vault weight).
Analyses on wall assemblages, not reported here, performed by varying the number of bond stones, size of blocks and cavities, disposition of stones in the wall shown that a correct disposition of bonding stones must avoid stacks of short stones which, by overturning, break up the entire assembly. Finally Fig. 6 reports a satisfactory conclusion: the wall, though slender and with many voids, with a suitable arrangement of stones can all the same support not only the total weight of the vault but also a service load equal to thirty-five per cent of the vault self weight.

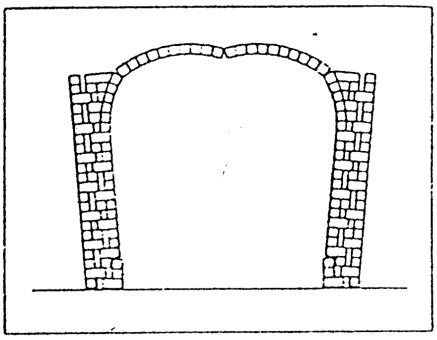

Fig. 6

8 CONCLUDING REMARKS

None of the procedures here discussed is free from disadvantages and limitations. Refinements should be introduced, for instance, in the LA code, to solve the numerical problems pointed out, implementing an optimization routine tailored on the problem and re-designing the code to save CPU space, but some limitations are intrinsic to the specific constitutive models and cannot be eliminated. A future development could be based on the co-operation of the different codes. The simplicity of use of FE codes could be taken advantage of, to obtain a first solution to be passed as input data to the LA procedure, aiming at the calculation of the minimum load factor interesting from the point of view of structural safety. When imperfect contacts must be taken into account, the ability of DE in finding contact points during the analysis could be used as well, in connection with a FE approach, to determine a modified structure following the imperfections of the contact surfaces; this one should be analysed (for the live loads) by a FE procedure, to save computer time. Anyway, all the discrete methods presented proved to be able to properly model systems of dry-assembled blocks and to correctly evaluate the failure mechanism and the load factor, pointing out, at the same time, the relevant influence of the brickwork texture in determining the ultimate strength of these structures.

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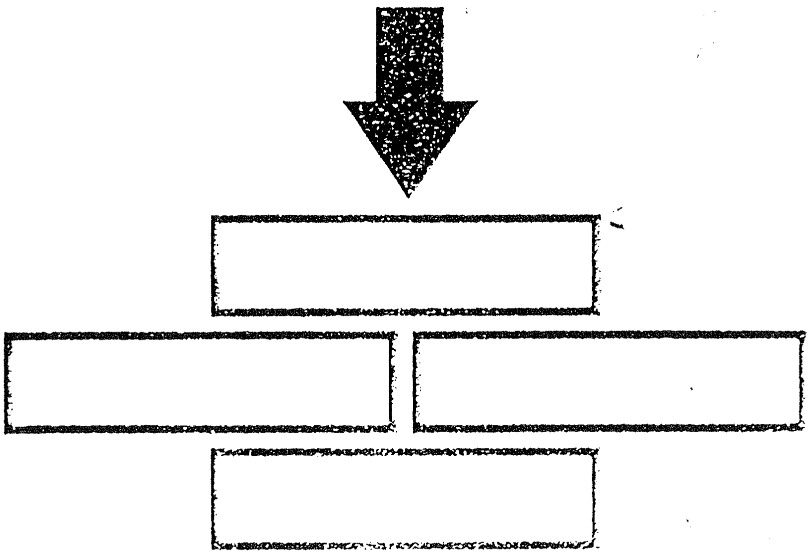

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